

COULOMB GAP IN THE QUANTUM HALL INSULATOR

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We calculate numerically the spectrum of disordered electrons in the lowest Landau level at filling factor $1/5$ using the self-consistent Hartree-Fock approximation for systems containing up to 400 flux quanta. Special attention is paid to the correct treatment of the $q = 0$ component of the Coulomb interaction. For sufficiently strong disorder, the system is an insulator at this filling factor. We observe numerically a Coulomb gap in the single-particle density of states (DOS). The DOS agrees quantitatively with the predictions for classical point charges.

The Coulomb interaction between charged localized particles leads to a vanishing single particle density of states (DOS) at the Fermi energy, $\rho(E)$. This classical effect was understood by Efros and Shklovskii as a necessary condition for the stability of the ground state with respect to particle-hole excitations¹. A more quantitative description of this Coulomb gap was obtained by Efros using a self-consistent method².

The argument leading to the Coulomb gap is expected to hold in a quantum mechanical system, too, if the localization length of the particles is small compared to their distance. An important example of such a system is a disordered two-dimensional electron gas in a strong magnetic field, exhibiting the integer quantum Hall effect³. In this system, all states are exponentially localized except those at single energies close to the centers of each Landau level⁴. Indeed, Yang and MacDonald numerically found a linearly vanishing DOS when the filling factor ν of the lowest Landau level was $1/5$ ⁵. Remarkably, the vanishing of the DOS persists even at the critical energy in the band center^{5,6}.

In this communication, we show that the DOS in the lowest Landau level at filling factor $1/5$ can be quantitatively described by the self-consistent method of Efros². We carefully consider the finite-size effects in the self-consistent Hartree-Fock (HF) approximation and apply Efros' ideas to the finite systems considered in the numerical simulations. The agreement is surprising, since the effects of wavefunction overlap and exchange interaction are present in the distribution of the HF matrix elements.

Before restricting our discussion to the QH system, we want to discuss the self-consistent Hartree-Fock approximation of spinless (or fully spinpolarized, as in the lowest Landau level) electrons on a finite torus of area $L \times L$. The Hamiltonian is

$$\mathcal{H} = \sum_{i=1}^N [T_i + U_{\text{dis}}(\hat{\mathbf{r}}_i) + V_{\text{el-bg}}(\hat{\mathbf{r}}_i)] + \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^N V(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j) + V_{\text{bg-bg}}, \quad (1)$$

where T_i is the kinetic energy, U_{dis} the disorder potential, $V_{\text{el-bg}}$ the interaction with the homogeneous background charge, V the Coulomb interaction, and $V_{\text{bg-bg}}$ the background self-interaction, respectively. The Hartree-Fock equations then take

the form

$$(T + U_{\text{dis}} + V_{\text{el-bg}})\psi_{\alpha}(\mathbf{r}) + \sum_{\beta} f_{\beta} \sum_{\mathbf{q}} [(\beta|\beta)_{\mathbf{q}}\psi_{\alpha}(\mathbf{r}) - (\beta|\alpha)_{\mathbf{q}}\psi_{\beta}(\mathbf{r})] v_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}} = \varepsilon_{\alpha}\psi_{\alpha}(\mathbf{r}), \quad (2)$$

with $(\beta|\alpha)_{\mathbf{q}} = \langle\beta|e^{-i\mathbf{q}\cdot\mathbf{r}}|\alpha\rangle$, $v_{\mathbf{q}} = 2\pi e^2/(L^2 q)$ is the two-dimensional Fourier transform of the Coulomb interaction, and f_{α} is the Fermi function. The $\mathbf{q} = \mathbf{0}$ part of the sum over \mathbf{q} gives

$$v_0 \sum_{\beta} f_{\beta} \psi_{\alpha}(\mathbf{r})(1 - \delta_{\alpha,\beta}) = v_0(N - f_{\alpha})\psi_{\alpha}(\mathbf{r}) = -V_{\text{el-bg}}\psi_{\alpha}(\mathbf{r}) - v_0 f_{\alpha}\psi_{\alpha}(\mathbf{r}), \quad (3)$$

which cancels the interaction with the background charge up to a contribution proportional to $v_0 = \int d^2r r^{-1} L^{-2} \sim L^{-1} < \infty$. Since this contribution vanishes in the limit of infinite system size it is often neglected resulting in the HF equations

$$(T + U_{\text{dis}})\psi_{\alpha}(\mathbf{r}) + \sum_{\beta, \mathbf{q} \neq \mathbf{0}} f_{\beta} [(\beta|\beta)_{\mathbf{q}}\psi_{\alpha}(\mathbf{r}) - (\beta|\alpha)_{\mathbf{q}}\psi_{\beta}(\mathbf{r})] v_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}} = \varepsilon_{\alpha}\psi_{\alpha}(\mathbf{r}). \quad (4)$$

However, the correct eigenvalues for the finite system would be

$$\tilde{\varepsilon}_i = \varepsilon_i - f_i v_0, \quad (5)$$

as an electron in an empty state interacts with N other electrons, while an electron in an occupied state interacts with only $N - 1$ other electrons.

Efros based his self-consistent method ² on the observation that the ground state of a system is stable against every particle-hole excitation. For the solutions ε_i of eq. (4) this is equivalent to the condition

$$\Delta E = \varepsilon_j - \varepsilon_i - v_{ij} > 0, \quad (6)$$

for every i, j . In the classical limit, for states localized in an infinitesimal vicinity of the position \mathbf{r}_i , the interaction matrix element is given by

$$v_{ij} = \sum_{\mathbf{q} \neq \mathbf{0}} v_{\mathbf{q}} (i|i)_{\mathbf{q}}(j|j)_{-\mathbf{q}} = \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|} - v_0. \quad (7)$$

For the torus geometry,

$$v_0 = \frac{e^2}{L^2} \int_{L \times L} d^2r \frac{1}{r} = 2 \ln(1 + \sqrt{2}) \frac{e^2}{L} \approx 1.76 \frac{e^2}{L}. \quad (8)$$

In the classical limit, eq. (6) restricts the possible distances $|\mathbf{r}_i - \mathbf{r}_j|$ and energies ε_j for a given energy ε_i . This leads to a self-consistency condition for the DOS ²

$$\rho(\varepsilon) = \rho_0 \exp \left[-\frac{\pi}{2} e^4 \int_0^{\infty} \frac{\rho(\varepsilon') d\varepsilon'}{(\varepsilon + \varepsilon' + v_0)^2} \right]. \quad (9)$$

In contrast to the infinite system result of Efros, eq. (9) in a finite system contains the finite contribution v_0 .

In the limit of small ε and large L , eq. (9) has the asymptotic solution

$$\rho(\varepsilon) = \frac{2}{\pi e^4} (v_0 + |\varepsilon|). \quad (10)$$

The stability condition for the true HF eigenvalues $\tilde{\varepsilon}_i$ (eq. (5)) is

$$\Delta E = \tilde{\varepsilon}_j - \tilde{\varepsilon}_i - v_{ij} - v_0 > 0. \quad (11)$$

The self-consistency equation for the density of the ε does not contain v_0 . However, now a lower bound for the energy difference $\tilde{\varepsilon}_j - \tilde{\varepsilon}_i$ exist, since the distance between the states can be at most $L/\sqrt{2}$ on the torus:

$$\rho(\tilde{\varepsilon}) = \rho_0 \exp \left[-\frac{\pi}{2} e^4 \int_{\sqrt{2}e^2/L}^{\infty} \frac{\rho(\tilde{\varepsilon}') d\tilde{\varepsilon}'}{(\tilde{\varepsilon} + \tilde{\varepsilon}')^2} \right]. \quad (12)$$

Again, the asymptotic solution is

$$\rho(\tilde{\varepsilon}) = \frac{2}{\pi e^4} \left(a\sqrt{2} \frac{e^2}{L} + |\varepsilon| \right), \quad (13)$$

where $a \approx 0.567$ is the solution of $\ln a = -a$.

We solved the self-consistent HF equations for lowest Landau level with a short-ranged disorder potential^{5,6}. The resulting DOS for a system containing 400 flux quanta at filling factor 1/5 averaged over 60 disorder realizations is shown in Fig. 1. In order to avoid a filling in of the gap due to fluctuations of the Fermi energy, the spectra were shifted to assure $\varepsilon_N + \varepsilon_{N+1} = 0$ for every disorder realization, where N is the number of electrons. Comparison with the self-consistent solution of eq. (9) and its asymptotic solution eq. (10) shows qualitative agreement. The deviations at larger distance from the Fermi energy are due to the assumption of a constant DOS in the derivation of eq. (9).

The DOS of the true HF eigenvalues $\tilde{\varepsilon}$ shows qualitatively the same behavior. In Fig. 2 a) the DOS at the Fermi energy is shown for different system sizes and disorder strength. The data depend only on $l_B/(\gamma L)$, where $\gamma = e^2/(l_B \Gamma)$ is a measure of the strength of the interactions relative to the disorder Γ , and for small arguments are well described by the asymptotic solution eq. (13).

The agreement between numerical simulations and the classical self-consistent method is surprising, as the conditions for its applicability are only rather poorly fulfilled in our system. The localization length $\xi \approx 4l_B$ of the non-interacting system is only slightly smaller than the mean particle spacing of $(4\pi/(3^{1/2}\nu))^{1/2}l_B \approx 6l_B$ ⁴. Thus one might expect to see stronger effects of the finite wavefunction overlap and the exchange interaction. Indeed, the distribution of interaction matrix elements shows these effects. Corresponding to a matrix element v_{ij} we define an effective distance $r_{\text{eff}} = e^2/(v_{ij} + v_0)$ and plot its histogram in Fig. 2 b). For a random distribution of classical point charges one would expect a distribution that rises linearly for small distances and that terminates at the maximum distance $L/\sqrt{2}$. In contrast, effective distances much larger than L are present corresponding to states with small interaction matrix elements due to the exchange interaction.

We studied numerically the Coulomb gap at low filling factor in an interacting, disordered model of the integer quantum Hall effect. Surprisingly, we find quantitative agreement with the predictions of Efros' classical self-consistent method, despite the fact that the distribution of interaction matrix elements shows strong effects of the quantum mechanical effects of wavefunction overlap and exchange.

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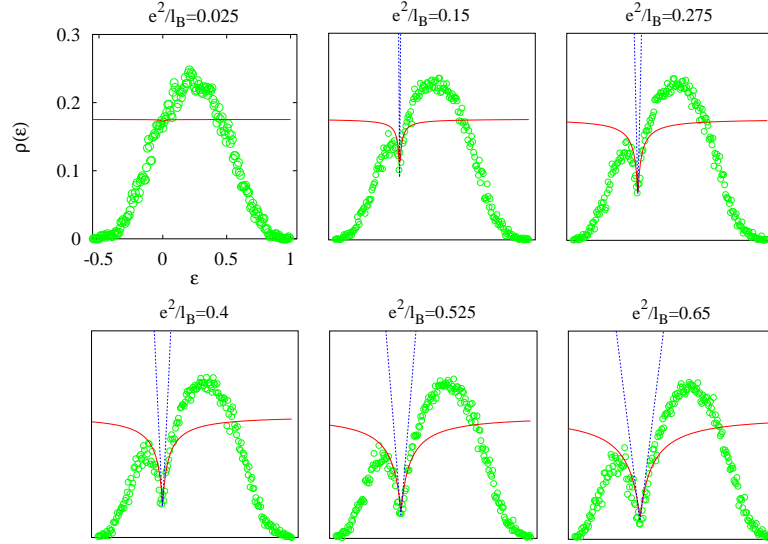


Figure 1. The density of states obtained by smearing every eigenvalue by a Gaussian of width 0.8 mean level spacings for interaction strengths $e^2/(l_B\Gamma)$ of 0.025, 0.15, 0.275, 0.4, 0.525, and 0.65 (top left to right bottom). The dashed lines correspond to the solution of eq. (9) and the asymptotic solution eq. (10).

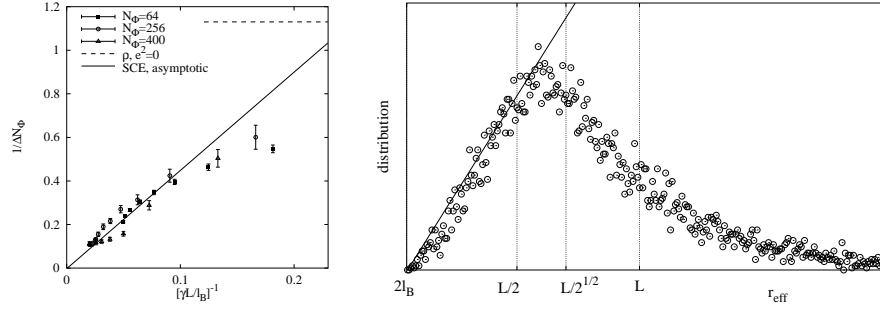


Figure 2. (a) The averaged DOS at the Fermi energy $\rho(0) = 1/(L^2(\epsilon_{N+1} - \epsilon_N))$ of the true HF eigenvalues as a function of $l_B/(\gamma L)$. (b) Distribution of effective distance r_{eff} of the HF states.

References

1. A.L. Efros and B.I. Shklovskii, *J. Phys. C* **8**, L49 (1975).
2. A. L. Efros, *J. Phys. C* **9**, 2021 (1976).
3. K. von Klitzing, G. Dorda, and M. Pepper, *Phys. Rev. Lett.* **45**, 494 (1980).
4. Bodo Huckestein, *Rev. Mod. Phys.* **67**, 357 (1995).
5. S.-R. Eric Yang and A. H. MacDonald, *Phys. Rev. Lett.* **70**, 4110 (1993).
6. Bodo Huckestein and Michael Backhaus, *Phys. Rev. Lett.* **82**, 5100 (1999).